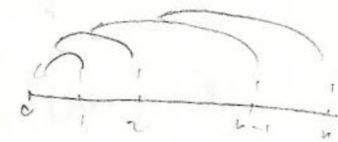
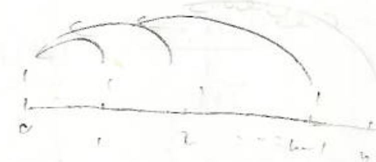
Tutorial 3.

Preliminary:

| | Accumulated Value | Present Value |
|---|---|---|
| Annuity-Immediate (end of the period) | $s_{\overline{n}\rvert i} = (1+i)^{n-1} + \cdots + (1+i) + 1 = \frac{(1+i)^n - 1}{i}$ | $a_{\overline{n}\rvert i} = v + v^2 + \cdots + v^n = \frac{1-v^n}{i}$ |
| Annuity-Due (beginning of the period) | $\ddot{s}_{\overline{n}\rvert i} = (1+i)^n + \cdots + (1+i)^2 + (1+i) = (1+i)s_{\overline{n}\rvert i} = \frac{(1+i)^n - 1}{d}$ | $\ddot{a}_{\overline{n}\rvert i} = 1 + v + \cdots + v^{n-1} = \frac{1-v^n}{1-v} = \frac{1-v^n}{d}$ |






2.1.1.

Option 1: Accumulate Value $= 50,000(1+i)^{24}$

Option 2: Annuity Payment is $K$, $K a_{\overline{24}\rvert 10\%} = 50,000 \Rightarrow K = 5564.99$

$K s_{\overline{24}\rvert 5\%} = 50,000(1+i)^{24} \Rightarrow i = 6.9\%$


$50,000 = K a_{\overline{24}\rvert 10\%} \Rightarrow K = 5564.99$

2.2.1.

(a) monthly rate $j = (1+i)^{\frac{1}{12}} - 1$, $i$ is annual rate,

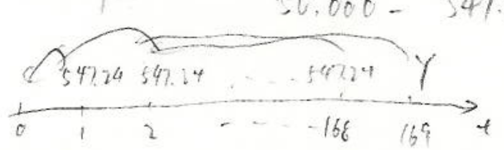
and $1+i = \left[1 + \frac{i^{(2)}}{2}\right]^2$, then $j = \left(\left(1+\frac{i^{(2)}}{2}\right)^2\right)^{\frac{1}{12}} - 1 = \left(1+\frac{i^{(2)}}{2}\right)^{\frac{1}{6}} - 1 = \left(1+\frac{10\%}{2}\right)^{\frac{1}{6}} - 1$

$X$ is monthly payment, $X a_{\overline{300}\rvert j} = 50,000$, $\Rightarrow X = 447.24$.

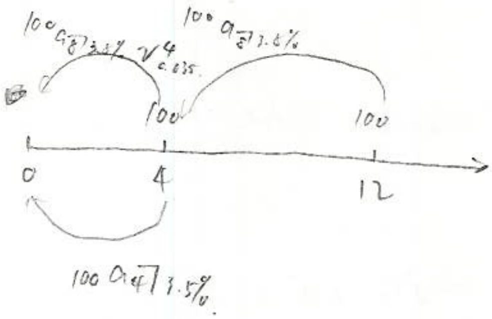
(b) $(X+100)a_{\overline{n}\rvert j} = 50,000 \Rightarrow a_{\overline{n}\rvert j} = 91.37$, $\Rightarrow n = 168.5$

$168^{th}$ payment occurs on Dec 31, 2023. by $X + 100 = 547.24$.

The fractional part $50,000 - 547.24 a_{\overline{168}\rvert j} = Y v_j^{169} \Rightarrow Y = 290.30$.



2.2.11.




$$1000 = 100\, a_{\overline{4}|0.035} + 100\, a_{\overline{8}|0.035} \cdot v^4_{0.035}$$

$$\Rightarrow a_{\overline{8}|i} = 7.26 \Rightarrow i = 2.708\%.$$

2.1.27.

a) $\ddot{a}_{\overline{n}|i} = \dfrac{1-v^n}{d} = \dfrac{1-v^n}{\frac{i}{1+i}} = (1+i)\dfrac{1-v^n}{i} = (1+i)a_{\overline{n}|i}^{*} = a_{\overline{n}|i} + i\,a_{\overline{n}|i} = a_{\overline{n}|i} + (1-v^n)^{*}$

$= v+v^2+\dots+v^n+1-v^n = 1+v+\dots+v^{n-1} = a_{\overline{n-1}|i}$.

b) $\ddot{s}_{\overline{n}|i} = \dfrac{(1+i)^n-1}{d} = \dfrac{(1+i)^n-1}{\frac{i}{1+i}} = (1+i)\cdot\dfrac{(1+i)^n-1}{i} = (1+i)s_{\overline{n}|i}^{*}$

$= s_{\overline{n}|i} + i\,s_{\overline{n}|i} = s_{\overline{n}|i} + (1+i)^n-1^{*} = (1+i)^{n-1}+\dots+(1+i)+1+(1+i)^n-1 = s_{\overline{n+1}|i}-1.$

Tutorial:

2.1.1.

2.2.1

2.2.11

2.1.27

Problem set:

2.1.4 2.2.2

2.1.5 2.2.7

2.1.7 2.2.8

2.1.9 2.2.11

2.1.12

2.1.33

2.1.4.

$1000\ddot{s}_{\overline{300}|.01} = Y\ddot{a}_{\overline{300}|0.01} \Rightarrow Y = \dfrac{1000\ddot{s}_{\overline{300}|0.01}}{\ddot{a}_{\overline{300}|0.01}} = 1000 \cdot \dfrac{\frac{(1+i)^{300}-1}{d}}{\frac{1-(\frac{1}{1+i})^{300}}{d}} = 1000(1+i)^{300}$,

$i = \dfrac{i^{(12)}}{12} = 0.01$, $\quad Y = 19,788.47$.


2.1.5.

i) $100\,s_{\overline{7}|0.0075} = 715.95$, $\quad \dfrac{i^{(12)}=9\%}{12} = 0.0075$

ii) $100\,s_{\overline{19}|0.0075} = 2033.87$, $\quad \dfrac{i^{(12)}=9\%}{12} = 0.0075$.

iii) $\quad i_1 = \dfrac{0.105}{12} = 0.00875$, $\quad i_2 = \dfrac{0.12}{12} = 0.01$,

$100\left[\,s_{\overline{19}|0.0075}\cdot(1.00875)^{9}(1.01)^{4} + s_{\overline{9}|0.00875}(1.01)^{4} + s_{\overline{4}|0.01}\,\right] = 3665.12$.

iv) $\quad 3665.12 \times 0.01 = 36.65$.

2.1.7.

a) $10(1.05)^{30}\cdot s_{\overline{10}|.05} + 20(1.05)^{20}s_{\overline{10}|.05} + 30(1.05)^{10}s_{\overline{10}|.05} + 40\,s_{\overline{10}|.05} = 2328.82$

b) $\cancel{10(s_{\overline{40}|.05} + s_{\overline{30}|.05} + s_{\overline{20}|.05} + s_{\overline{10}|.05})}$

$10\,[\,s_{\overline{40}|} - s_{\overline{30}|} + 2(s_{\overline{30}|} - s_{\overline{20}|}) + 3(s_{\overline{20}|} - s_{\overline{10}|}) + 4s_{\overline{10}|}\,] = 10(s_{\overline{10}|.05} + \cdots + s_{\overline{40}|.05})$.

$s_{\overline{40}|} - s_{\overline{30}|} = (1.05)^{30}\cdot s_{\overline{10}|} \Rightarrow \dfrac{(1+i)^{40}-1}{i} - \dfrac{(1+i)^{30}-1}{i} = (1+i)^{30}\cdot\dfrac{(1+i)^{10}-1}{i}$

2.1.9.

$I_t = 1\cdot i\cdot s_{\overline{t-1}|i} = i\cdot\dfrac{(1+i)^{t-1}-1}{i} = (1+i)^{t-1}-1$

$\sum_{t=1}^{n} I_t = \sum_{t=1}^{n}\left[(1+i)^{t-1}-1\right] = \sum_{t=1}^{n}(1+i)^{t-1} - n = \dfrac{(1+i)^{n}-1}{i} - n = s_{\overline{n}|i} - n$.

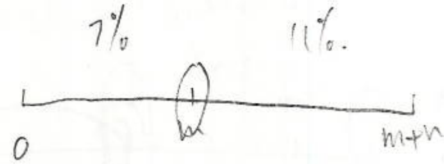
Total interest = total accumulated value − total deposit.

$1\cdot s_{\overline{m}|7\%}\cdot(1+11\%)^{n}$

2.1.12.

$A\text{-}V = 1\cdot s_{\overline{m}|0.07}\cdot(1+11\%)^{n} + 1\cdot s_{\overline{n}|0.11} = 128 + 34\cdot(1.11)^{n}$

$s_{\overline{n}|0.11} = \dfrac{(1.11)^{n}-1}{0.11} = 128, \Rightarrow (1.11)^{n} = 15.08, \Rightarrow A\text{-}V = 640.72$.



2.1.33.

(a)

$v^t \cdot s_{\overline{n}|} = (1+i)^{-t} \cdot \frac{(1+i)^n - 1}{i} = \frac{(1+i)^{n-t} - (1+i)^{-t}}{i}$,

$a_{\overline{t}|} = \frac{1-v^t}{i} = \frac{1-(1+i)^{-t}}{i}$, $s_{\overline{n-t}|} = \frac{(1+i)^{n-t} - 1}{i}$,

$\ddot{a}_{\overline{t+1}|} = (1+i)\, a_{\overline{t+1}|}$, $\ddot{s}_{\overline{n-t+1}|} = (1+i)\, s_{\overline{n-t+1}|}$

$\ddot{a}_{\overline{t+1}|} + \ddot{s}_{\overline{n-t+1}|} = (1+i) \cdot \frac{1-(1+i)^{-t-1}}{i} + (1+i) \cdot \frac{(1+i)^{n-t+1} - 1}{i}$

(b)

$(1+i)^t \ddot{a}_{\overline{n}|} = (1+i)^t \cdot (1+i)\, a_{\overline{n}|} = (1+i)^{t+1} \cdot \frac{1-(1+i)^{-n}}{i} = \frac{(1+i)^{t+1} - (1+i)^{t+1-n}}{i}$

$\ddot{s}_{\overline{t}|} + \ddot{a}_{\overline{n-t}|} = (1+i)\, s_{\overline{t}|} + (1+i)\, a_{\overline{n-t}|} = (1+i) \cdot \frac{(1+i)^t - 1}{i} + (1+i) \cdot \frac{1-(1+i)^{t-n}}{i}$

$s_{\overline{t+1}|} + a_{\overline{n-t+1}|} = \frac{(i+1)^{t} - 1}{i} + \frac{1-(1+i)^{-(n-t+1)}}{i}$.

2.2.2

Derek: $1200\, \ddot{s}_{\overline{25}|0.06} = 1200(1+0.06)\, s_{\overline{25}|0.06} = 1200 \times 1.06 \times \frac{(1.06)^{25} - 1}{0.06} = 69.787.66$

Anne: $1200\, s_{\overline{25}|0.06} = \frac{69.787.66}{1.06} = 65.837.41$

Ira: $100\, \ddot{s}_{\overline{300}|j}$ $j = (1.06)^{\frac{1}{12}} - 1$. $100\, \ddot{s}_{\overline{300}|j} = 67.958.10$

2.2.7.

~~42000~~

$10,000(1.05)^n \cdot 0.05 \geq 2000 \Rightarrow n \geq 28.4$.

3800

At time 29 $10,000(1.05)^{29} = 41,161.36$.

1900.

smaller scholarship $1,161.36$.

38000 + 1900

$(10,000(1.05)^n - 2000) \cdot 0.05 \geq 2000$,

2.2.8.

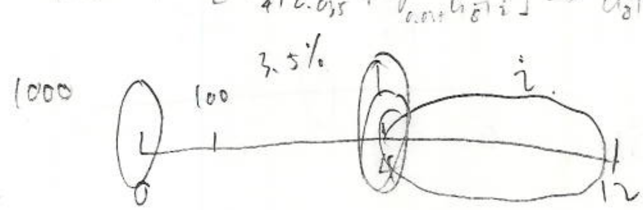
$i = (1.0075)^{12} - 1 = 0.09$.

$AV = 100\, \ddot{s}_{\overline{12n}|j} + 1000\, s_{\overline{n}|i} \geq 100,000 \Rightarrow \frac{(1+i)^n - 1}{d_j} + 10 \cdot \frac{(1+i)^n - 1}{i} \geq 1000$

$\Rightarrow n \geq 18.3$. $n = 19$. $\ddot{s}_{\overline{228}|j} + 10\, s_{\overline{19}|i} = 1078.2$. April 30th.

2.2.11.

$1000 = 100\,[a_{\overline{4}|0.035} + v^4_{0.035}\, a_{\overline{8}|i}] \Rightarrow a_{\overline{8}|i} = 7.26. \Rightarrow i = 2.708\%$.




$100\, a_{\overline{8}|i} \cdot v^4 + 100 \cdot a_{\overline{4}|3.5\%} = 1000$